Chapter 5
Section 5.2

Let α be an angle whose initial side is the positve x-axis. Assume that the terminal side of α hits the unit circle at the point (x, y). We will give definitions to this point in order to reference it more easily.

Def: If α is an angle and (x, y) is the point of intersection of the terminal side and the unit circle, then

$$\sin(\alpha) = y \text{ and } \cos(\alpha) = x$$

Note: In the textbook, the parentheses are sometimes omitted but remember that you are putting the angle α into the function *sine* or *cosine*.

Domain: The domain of the *sine* and *cosine* functions are all the possible angles. Since we can make an angle with any measure the domain of *sine* and *cosine* is the set of real numbers$=\mathbb{R} = (-\infty, \infty)$.

Range: Since the *sine* and *cosine* functions measure a point on the unit circle, the range of each function is $[-1, 1]$.

Grp Ex: Find the exact values of *sine* and *cosine* for

a)$90°$ $\quad \sin(90)=1 \quad \cos(90)=0$

b)$\frac{\pi}{2}$ $\quad \sin(\pi/2)=1 \quad \cos(\pi/2)=0$

c)$180°$ $\quad \sin(180)=0 \quad \cos(180)=-1$

d)$-\frac{5\pi}{2}$ $\quad \sin(-\frac{5\pi}{2})=-1 \quad \cos(-\frac{5\pi}{2})=0$

Multiples of $45° = \frac{\pi}{2}$ rad

Note that the terminal side of a $45°$ angle lies on the line $y = x$. So the x value on the unit circle at its intersection is equal to the y value at the intersection.

Ex: Find $\sin(45°)$

$$\sin(45°)=\frac{\sqrt{2}}{2}$$

Multiples of $30° = \frac{\pi}{6}$ rad

Note that if our angle has a measure of $30°$ it creates a right triangle with the initial side. This triangle is a special $30-60-90$ triangle. So one leg is twice the length of the other leg.

Ex: Find $\sin(30°)$ and $\cos(60°)$

$$\sin(30°)=\frac{1}{2} \qquad \cos(60°)=\frac{1}{2}$$

The Unit Circle

The unit circle is so easy to work with for two reasons. The obvious one, that comes from the definition of the unit circle, is that all points lie exactly 1 unit from the origin. The second is that the circle can be reflected across any line through the origin and remain the same. In particular, the x and y axes.

Group Work: Fill out the following unit circle on the additional handout using the symmetry of the unit circle.

Def: If θ is a nonquadrantal angle (not a mulitple of 90°), then the **reference angle** for θ is the positive angle γ formed by the terminal side of θ and the positive or negative x-axis.

Ex: Find the reference angle γ for $\theta = 120°$ and $\beta = \frac{7\pi}{6}$

$\gamma = 180° - 120° = 60°$ for θ

$\gamma = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$ for β

Thm: For any angle θ, the value of a trigonmetric function of θ can be found by finding the value of the function for its reference angle γ and prefixing the appropriate sign.

Ex: Find $\sin(\frac{7\pi}{4})$ and $\cos(-\frac{\pi}{6})$

$\sin(\frac{7\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.

The Fundamental Identity: If α is any angle or real number,

$$\sin^2(\alpha) + \cos^2(\alpha) = (\sin(\alpha))^2 + (\cos(\alpha))^2 = 1$$

Ex: Find $\cos(\alpha)$ if $\sin(\alpha) = 3/5$ and α is an angle in quadrant II.

$1 = \cos^2\alpha + \sin^2\alpha = \cos^2\alpha + (3/5)^2$

$1 = \cos^2\alpha + \frac{9}{25}$

$1 - \frac{9}{25} = \cos^2\alpha$

$\pm\frac{4}{5} = \pm\sqrt{\frac{16}{25}} = \cos\alpha$

Since $\cos\alpha \leq 0$ in Quadrant II, $\cos\alpha = -\frac{4}{5}$.

Practice: 8, 13, 18, 40, 43, 50, 55, 94